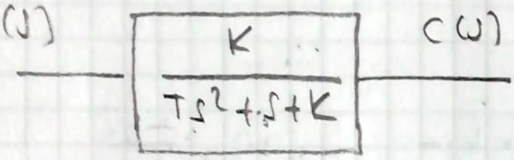
Soru:

Bir kontrol sisteminde çıkış ile giriş arasındaki transfer fonksiyonu aşağıda verilmiştir. Bu sistemde maksimum aşma %25,4 ve zirve zamanı 3 sn olması için K ve T katsayılarını hesaplayınız.

$$G(s) = \frac{K}{Ts^2 + s + K}$$



$$M_p = e^{\frac{-\pi \cdot \xi}{\sqrt{1-\xi^2}}}$$

$$\ln(0{,}254) = \ln e^{\frac{-3{,}14 \cdot \xi}{\sqrt{1-\xi^2}}} \Rightarrow \ln(0{,}254) = \frac{-3{,}14 \cdot \xi}{\sqrt{1-\xi^2}}$$

$$-1{,}37 \cdot \sqrt{1-\xi^2} = -3{,}14 \cdot \xi$$

$$1{,}88 \cdot (1-\xi^2) = 9{,}86\,\xi^2$$

$$1{,}88 - 1{,}88 \cdot \xi^2 = 9{,}86\,\xi^2 \rightarrow 1{,}88 = 11{,}74\,\xi^2$$

$$\xi^2 = 0{,}16 \qquad \xi = \sqrt{0{,}16} = 0{,}4$$

$$t_p = \frac{\pi}{W_d} \Rightarrow 3 = \frac{\pi}{W_d} \qquad W_d = \frac{3{,}14}{3} = 1{,}046 \text{ rad/s.}$$

$$W_d = W_n \sqrt{1-\xi^2} \rightarrow 1{,}046 = W_n \cdot \sqrt{1-0{,}4^2}$$

$$W_n = 1{,}14 \text{ rad/s}$$

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+\underbrace{K}} = \frac{K/T}{s^2+\frac{1}{T}s+\underbrace{\frac{K}{T}}}$$

$$w_n^2 = \frac{K}{T} \Rightarrow w_n = \sqrt{\frac{K}{T}} \qquad G(s) = \frac{w_n^2}{s^2+2w_n\xi s+w_n^2}$$

$$2 \cdot \xi \cdot w_n = \frac{1}{T}$$

$$T = \frac{1}{2\xi w_n} = \frac{1}{2*0{,}4*1{,}14} = 1{,}09$$

$$\boxed{T = 1{,}09}$$

$$w_n = \sqrt{\frac{K}{T}} \Rightarrow w_n^2 = \frac{K}{T} \Rightarrow K = w_n^2 \cdot T$$

$$K = (1{,}09) \cdot (1{,}14)^2 = 1{,}42$$

$$\boxed{K = 1{,}42}$$

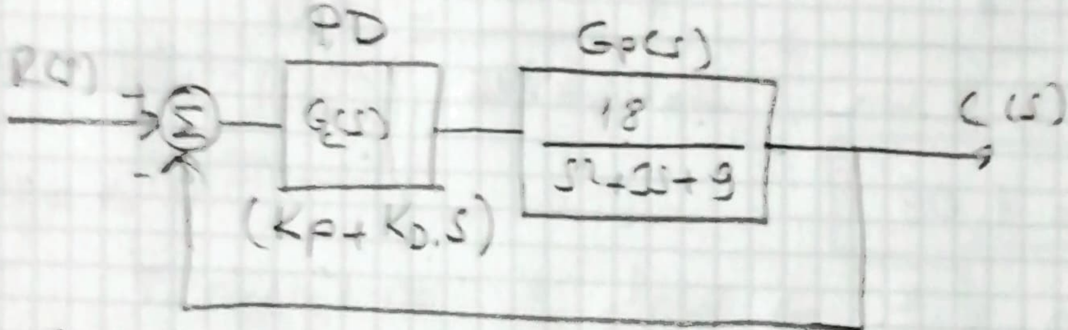
$$\frac{C(s)}{R(s)} = \frac{1{,}42}{1{,}09s^2+s+1{,}42}$$

Soru:

Kontrol edilecek sistemin (plant) transfer fonksiyonu

$G(s) = \dfrac{18}{s^2+3s+9}$ olarak verilmiştir.

Bu sisteme PD kontrolör kullanarak en fazla aşma (maksimum aşma) %=10 ve kalıcı durum hatası 0,1 olacak şekilde kontrol edilmek isteniyor. Bu koşulları sağlayan K_P ve K_D katsayılarını hesaplayınız.



$$G_c(s) = K_P + K_D.s$$

$$\frac{C(s)}{R(s)} = \frac{G_c(s)\,G_P(s)}{1+G_c(s).G_P(s)}$$

$$= \frac{(K_P + K_D s)\left(\dfrac{18}{s^2+3s+9}\right)}{1+\dfrac{18.K_P+18.K_D s}{s^2+3s+9}}$$

$$\frac{\dfrac{18K_P + 18K_D.s}{\cancel{s^2+3s+9}}}{\dfrac{s^2+3s+9+18K_D s+18K_P}{\cancel{s^2+3s+9}}}$$

$$G_K(s) = \frac{18K_D.s + 18K_P}{s^2 + \underbrace{(18K_D+3)}s + \underbrace{18K_P+9}_{w_n^2}}$$ → Kapalı Çevrim Transfer Fonksiyonu

$$M_P = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \Rightarrow \ln(0,1) = \ln\left(e^{\frac{-3,14\ \xi}{\sqrt{1-\xi^2}}}\right)$$

$$-2,3 = \frac{-3,14.\ \xi}{\sqrt{1-\xi^2}} \Rightarrow (0,73)^2 = \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)^2$$

$$0,53.(1-\xi^2) = \xi^2$$

$$0,53 - 0,53\xi^2 = \xi^2 \Rightarrow 0,53 = 1,53\,\xi^2$$

$$\xi^2 = \frac{0,53}{1,53} \Rightarrow \xi = 0,59$$

$$e(\infty) = \frac{1}{1 + \lim_{s\to 0} G_T(s)}$$

$$G_T(s) = G_c(s).\ G_P(s)$$

$$G_T(s) = \frac{18(K_P + K_D s)}{s^2+3s+9} = \frac{18\,K_D.s + 18K_P}{s^2+3s+9}$$

$$e(\infty) = 0,1 = \frac{1}{1+\frac{18K_P}{9}}$$

$$0,1 = \frac{1}{1+2K_P} \Rightarrow 0,1 + 0,2K_P = 1 \quad \boxed{K_P = 4,5}$$

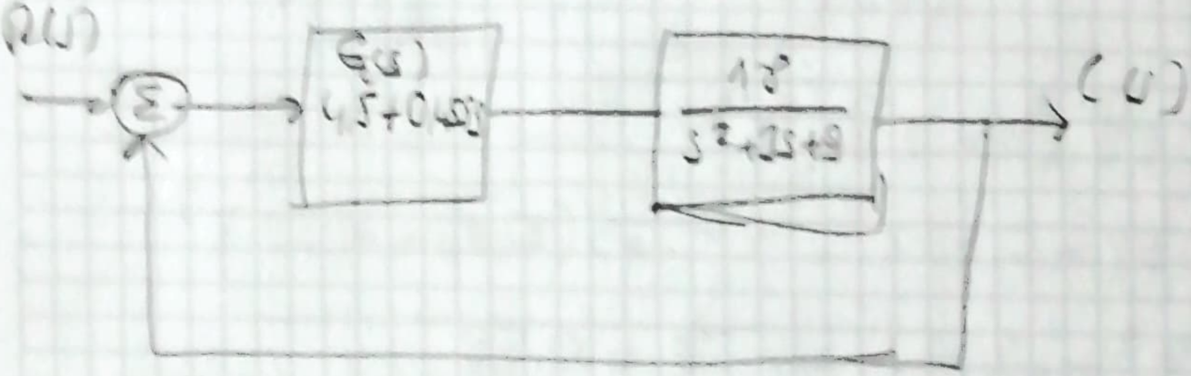
$18K_p + 9 = W_n^2$

$W_n^2 = 90 \Rightarrow W_n = 9{,}48$ rad/s.

$2 \cdot W_n \cdot \xi = 18K_D + 3$

$2 \cdot 9{,}48 \cdot 0{,}59 = 18 \cdot K_D + 3$

$K_D = 0{,}455$



$G_c(s) = \underbrace{4{,}5}_{K_p} + \underbrace{0{,}455}_{K_D} s$